Introduction to QCD

Lectures 5 and 6

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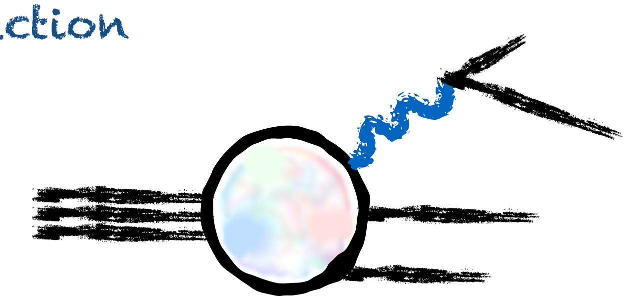
HUGS 2016

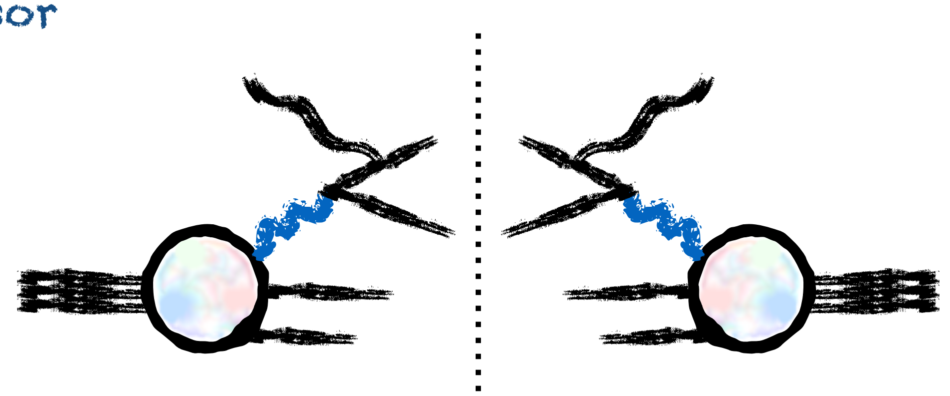
Jefferson Lab

Overview of the roadmap

1. We want to calculate this correction



2. We calculate hadronic tensor



$$W_{\mu\nu} = \frac{1}{4\pi M}\int_x^1 \frac{dy}{y} g(y) \tilde{W}_{\mu\nu}$$

3. We compare result with the form

$$F_2(x, Q^2) = \sum_i Q_i^2 x \left\{ q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right\}$$

4. Extract quark distribution functions with gluon correction

5. To simplify calculation we compute

$$W_T \equiv -g^{\mu\nu} W_{\mu\nu}$$

$$W_L \equiv P^\mu P^\nu W_{\mu\nu}$$

6. The last result we obtained was

$$\tilde{W}_T = 2Q_i^2 g^2 \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\frac{z}{1-z}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$\times \int_0^1 dv (1-v)^{-\epsilon} v^{-\epsilon} \left\{ (1-\epsilon)\left(\frac{1-v}{v} + \frac{v}{1-v}\right) - 2z(1-z)\frac{1}{v(1-v)} - 2\epsilon \right\}$$

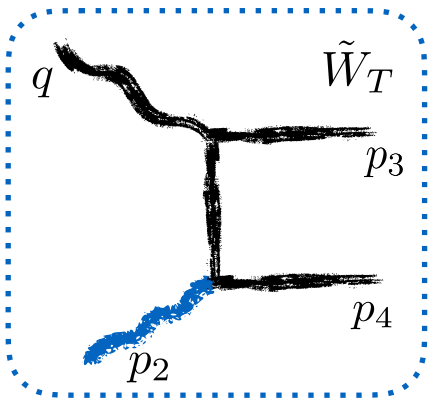
Transverse structure function

$$\tilde{W}_T = 2Q_i^2 g^2 \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\frac{z}{1-z}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}$$

$$\times \int_0^1 dv(1-v)^{-\epsilon}v^{-\epsilon}\left\{(1-\epsilon)\left(\frac{1-v}{v}+\frac{v}{1-v}\right)-2z(1-z)\frac{1}{v(1-v)}-2\epsilon\right\}$$

$$B(\mu,\nu) = \int_0^1 dx x^{\mu-1}(1-x)^{\nu-1}$$

To take angular integral we need beta function



$$-\frac{1}{\epsilon}\left\{2\frac{(1-\epsilon)^2}{1-2\epsilon}-4z(1-z)+\frac{2\epsilon^2}{1-2\epsilon}\right\}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

Collinear divergence

Expansion at d~4

$$-\frac{2}{\epsilon}\left(z^2+(1-z)^2\right)\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

We also should expand

$$\left(\frac{4\pi}{Q^2}\frac{z}{1-z}\right)^{\epsilon} = 1+\epsilon\ln\frac{4\pi}{Q^2}\frac{z}{1-z}+\dots$$

If we combine everything together

$$\tilde{W}_T = 2Q_i^2\frac{g^2}{4\pi}\left(z^2+(1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}-\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\ln\frac{4\pi}{Q^2}\frac{z}{1-z}\right\}$$

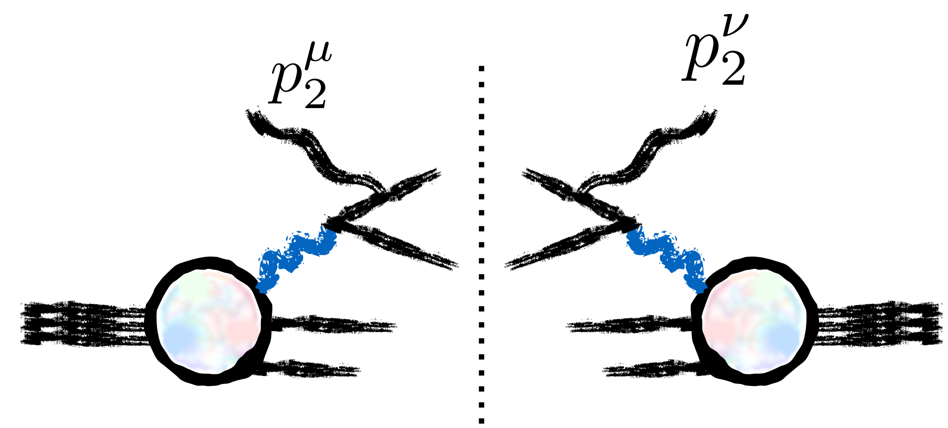
Cann't eliminate this, why?

Can eliminate this

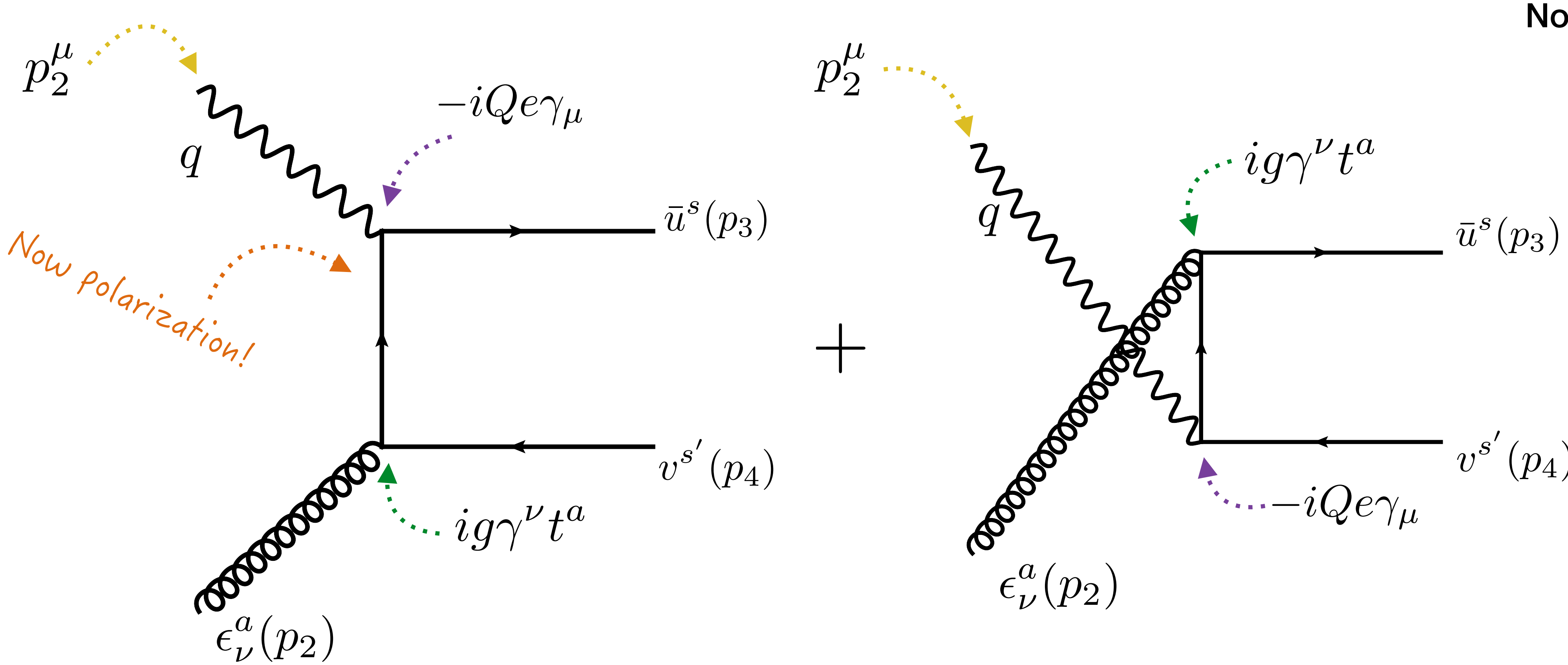
Longitudinal structure function

$$W_L = \frac{1}{4\pi M}\int_x^1 \frac{dy}{y^3} g(y)\tilde{W}_L$$

Note the different power



Now we have to perform calculation



$$p_2^\mu M_\mu = Q_i e g \bar{u}^s(p_3)\left[\not{p}_2 \frac{\not{p}_2 - \not{p}_4}{(p_2-p_4)^2}\gamma^\nu t^a + \gamma^\nu t^a \frac{\not{p}_3 - \not{p}_2}{(p_3-p_2)^2}\not{p}_2\right] v^{s'}(p_4) \times \epsilon_\nu^a(p_2)$$

Longitudinal structure function

1

$$p_2^\mu M_\mu = Q_i e g \bar{u}^s(p_3)\left[\not{p}_2 \frac{\not{p}_2 - \not{p}_4}{(p_2-p_4)^2}\gamma^\nu t^a + \gamma^\nu t^a \frac{\not{p}_3 - \not{p}_2}{(p_3-p_2)^2}\not{p}_2\right] v^{s'}(p_4) \times \epsilon_\nu^a(p_2)$$

2

$$\not{p}_2 \not{p}_2 = p_2^2 = 0 \quad \longrightarrow \quad p_2^\mu M_\mu = Q_i e g \bar{u}^s(p_3)\left[\not{p}_2 \frac{-\not{p}_4}{(p_2-p_4)^2}\gamma^\nu t^a + \gamma^\nu t^a \frac{\not{p}_3}{(p_3-p_2)^2}\not{p}_2\right] v^{s'}(p_4) \times \epsilon_\nu^a(p_2)$$

3

$$\not{p}_4 v(p_4) = 0 \qquad \bar{u}(p_3)\not{p}_3 = 0$$

It is pretty long but easy algebra

4

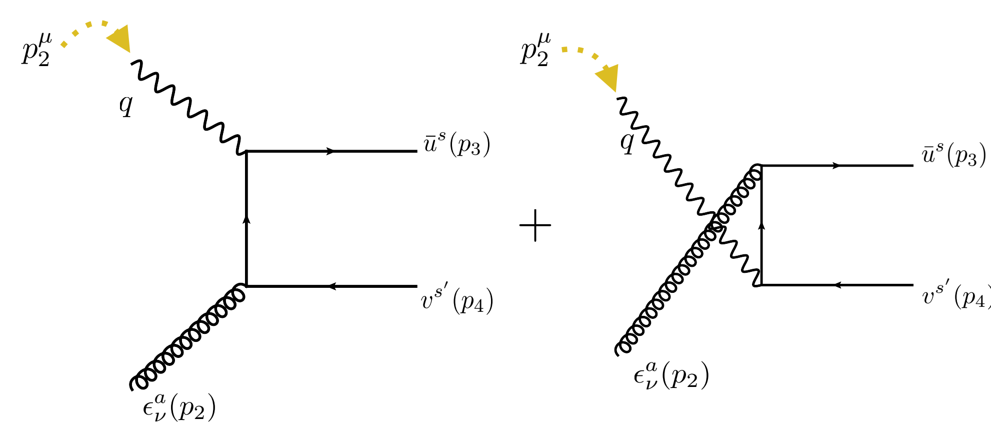
$$p_2^\mu M_\mu = Q_i e g \bar{u}^s(p_3)\left[-\frac{2p_4^\nu \not{p}_2}{t} + \frac{2p_3^\nu \not{p}_2}{u}\right] t^a v^{s'}(p_4) \times \epsilon_\nu^a(p_2)$$

No contribution

$-g_{\nu\sigma}$

5

$$p_2^\nu M_\nu^* = Q_i e g \bar{v}^{s'}(p_4)\left[-\frac{2p_4^\sigma \not{p}_2}{t} + \frac{2p_3^\sigma \not{p}_2}{u}\right] t^b u^s(p_3) \times \epsilon_\sigma^{*b}(p_2)$$



6

$$\sum_{\text{spin, color}} p_2^\mu p_2^\nu M_\mu M_\nu^* = 8 Q_i^2 e^2 g^2 \frac{p_3 \cdot p_4}{tu} Tr\{t^a t^a\} \times Tr\{\not{p}_3 \not{p}_2 \not{p}_4 \not{p}_2\}$$

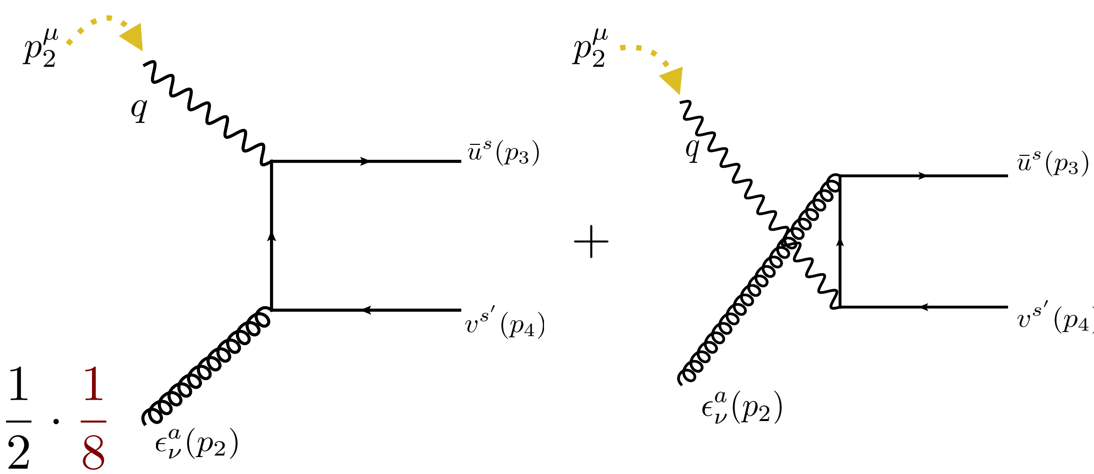
Looks like there is collinear singularity

Longitudinal structure function

6

$$\sum_{\text{spin, color}} p_2^\mu p_2^\nu M_\mu M_\nu^* = 8Q_i^2 e^2 g^2 \frac{p_3 \cdot p_4}{tu} Tr\{t^a t^a\} \times Tr\{\not{p}_3 \not{p}_2 \not{p}_4 \not{p}_2\}$$

Let's look more carefully



$\frac{1}{2} \cdot \frac{1}{8}$

7

$$\sum_{\text{spin, color}} p_2^\mu p_2^\nu M_\mu M_\nu^* = 8Q_i^2 e^2 g^2 \times 4s$$

No collinear divergence in W_L

We don't need dimensional reg.

8

$$\frac{1}{2}\frac{1}{8} \sum_{\text{spin, color}} p_2^\mu p_2^\nu M_\mu M_\nu^* = 2Q_i^2 e^2 g^2 s$$

No angular dependence

9 ?

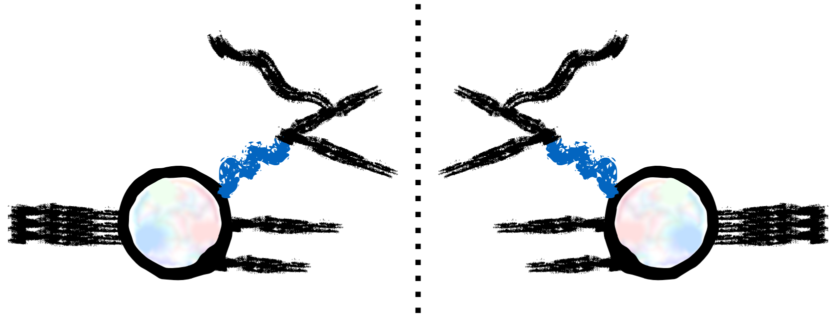
$$\int \frac{d^{d-1}p_3}{(2\pi)^{d-1}2E_3} \frac{d^{d-1}p_4}{(2\pi)^{d-1}2E_4} (2\pi)^d \delta^d(q + p_2 - p_3 - p_4) = \frac{1}{8\pi} \left(\frac{4\pi}{s}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int_0^1 dv (1-v)^{-\epsilon} v^{-\epsilon}$$

10

$$\tilde{W}_L = Q_i^2 \frac{g^2}{4\pi} Q^2 \frac{1-z}{z}$$

Longitudinal structure function

Structure function



We've calculate longitudinal and transverse structure functions for scattering on a single gluon

$$\tilde{W}_T = 2Q_i^2 \frac{g^2}{4\pi}\left(z^2 + (1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} - \ln\frac{4\pi}{Q^2}\frac{z}{1-z}\right\}$$

$$\tilde{W}_L = Q_i^2 \frac{g^2}{4\pi} Q^2 \frac{1-z}{z}$$

It is straightforward to obtain this function for scattering on a hadron

$$W_T = \frac{1}{4\pi M}\int_x^1 \frac{dy}{y} g(y)\tilde{W}_T \qquad W_L = \frac{1}{4\pi M}\int_x^1 \frac{dy}{y^3} g(y)\tilde{W}_L$$

Gluon distribution function

The form factor is given by a formula

$$(1-\epsilon)\frac{1}{M}F_2 = xW_T + 4\frac{x^3}{Q^2}(3-2\epsilon)W_L$$

For the form factor we get

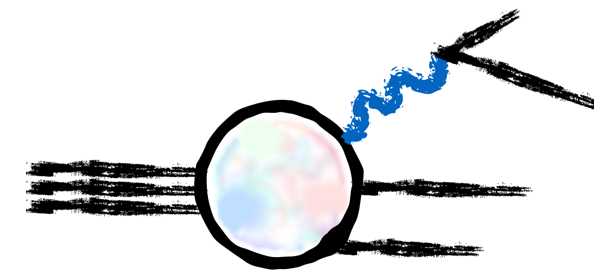
Almost final formula

$$(1-\epsilon)F_2 = \sum_i Q_i^2 \frac{g^2}{8\pi^2}\int_x^1 dy g(y) z$$
$$\times\left[\left(z^2 + (1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} - \ln\frac{4\pi}{Q^2}\frac{z}{1-z}\right\} + 6z(1-z)\right]$$

Correction to the quark distribution

Almost final formula

$$(1-\epsilon)F_2 = \sum_i Q_i^2 \frac{g^2}{8\pi^2}\int_x^1 dy g(y) z\left[\left(z^2+(1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} - \ln\frac{4\pi}{Q^2}\frac{z}{1-z}\right\} + 6z(1-z)\right]$$



Compare!

Alternatively we can define quark distribution function as

$$F_2(x,Q^2) = \sum_i Q_i^2 x\left\{q_i(x,Q^2) + \bar{q}_i(x,Q^2)\right\}$$

This part comes from here

Final formula!!!

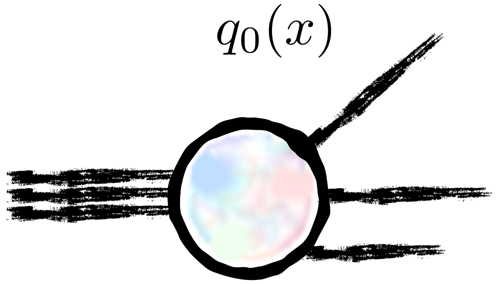
$$q(x,Q^2) = q_0(x) + \frac{g^2}{16\pi^2}\frac{1}{1-\epsilon}\int_x^1 \frac{dy}{y} g(y)\left[\left(z^2+(1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} + \ln\frac{Q^2}{4\pi}\frac{1-z}{z}\right\} + 6z(1-z)\right]$$

Finite function

Distribution function in the leading order (see, Lecture 2)

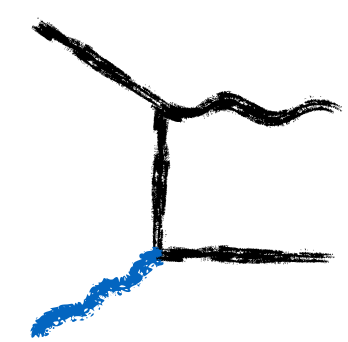
Explicit dependence on Q. Scaling violation.

$q_0(x)$



No dependence on Q!

Let's combine with parsonic cross section correction

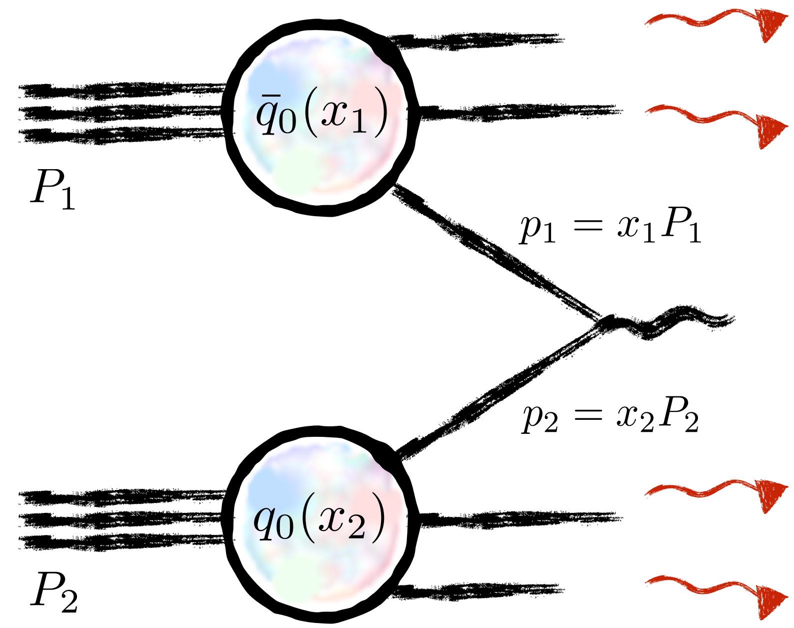


Correction to PDF

Let's update our result for the cross section in the leading order

$$d\sigma_0 = \frac{\pi e^2}{3}\frac{1-\epsilon}{S}\sum_i Q_i^2 \int_{\tau_0}^1 \frac{dx}{x}\left[\bar{q}_{0i}(x)q_{0i}(\frac{\tau_0}{x}) + q_{0i}(x)\bar{q}_{0i}(\frac{\tau_0}{x})\right]$$

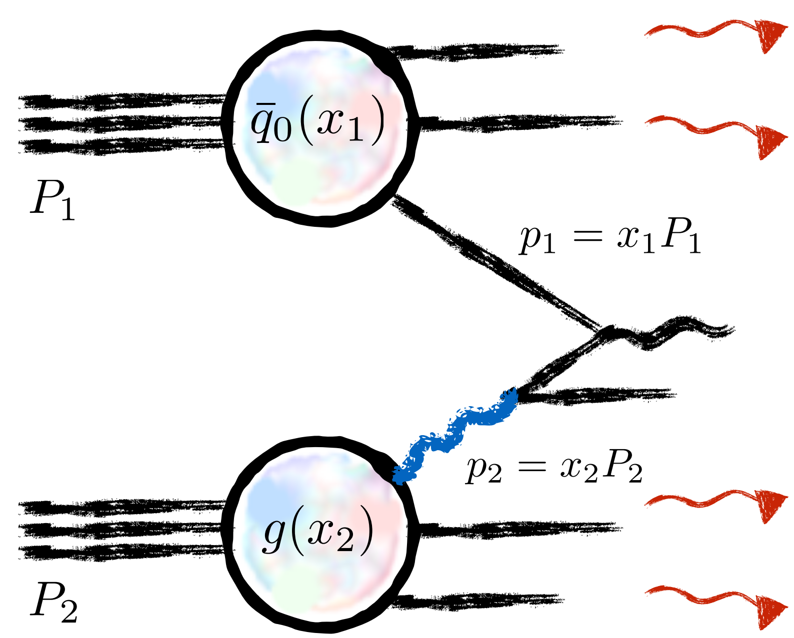
Take this function from the previous slide



$$d\sigma_1 = -\frac{e^2}{3S}\frac{g^2}{16\pi}\sum_i Q_i^2 \int_{\tau_0}^1 \frac{dx_1}{x_1}\int_{\tau_0/x_1}^1 \frac{dx_2}{x_2}\bar{q}_{0i}(x_1)g(x_2)\left[\left(z^2+(1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} + \ln\frac{Q^2}{4\pi}\frac{1-z}{z}\right\} + 6z(1-z)\right]$$

Note the minus sign

$$z = \frac{\tau_0}{x_1 x_2}$$

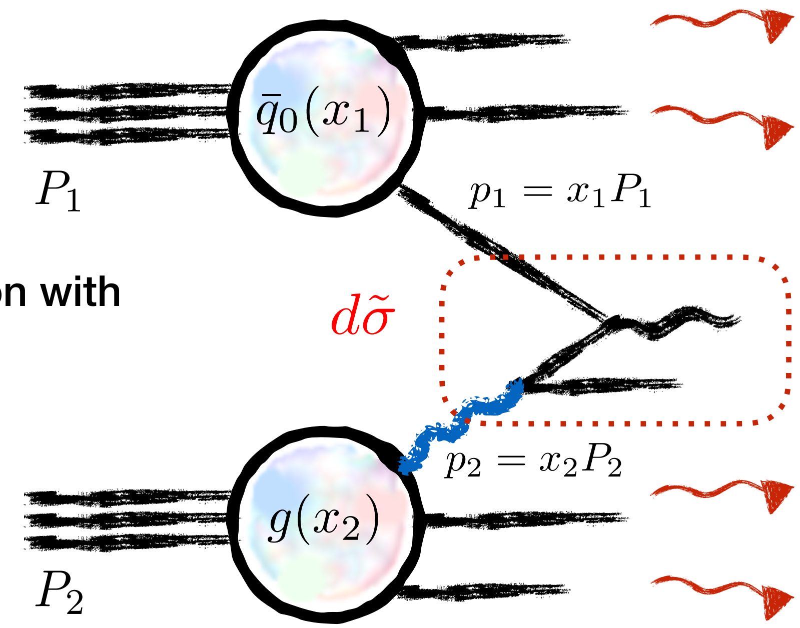


Correction to the partonic cross section

We start from the general factorization formula

$$d\sigma_1 = \sum_i \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}_{0i}(x_1) g(x_2) d\tilde{\sigma}$$

partonic cross section with radiative correction



Result from Lecture 3

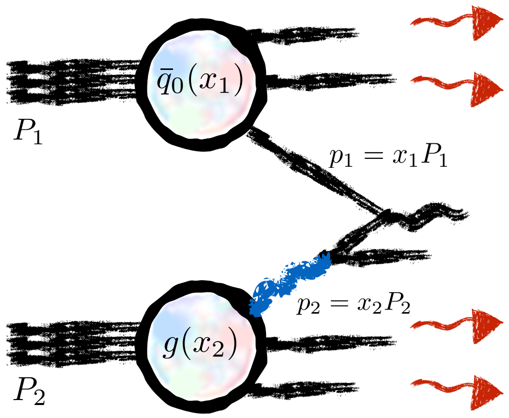
$$d\tilde{\sigma} = \frac{e^2 Q_i^2 g^2}{48\pi s} \times \left\{ \left(-\frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} - \ln \frac{4\pi}{M^2} \frac{M^2/s}{(1-M^2/s)^2} \right) \left[\frac{M^4}{s^2} + \left(1 - \frac{M^2}{s}\right)^2 \right] + \frac{3}{2} + \frac{M^2}{s} - \frac{3}{2}\frac{M^4}{s^2} \right\}$$

Substitution yields:

Recognize $z = \dfrac{M^2}{s}$

$$d\sigma_1 = \frac{e^2 g^2}{48\pi S} \sum_i Q_i^2 \int_{\tau_0}^{1} \frac{dx_1}{x_1} \int_{\tau_0/x_1}^{1} \frac{dx_2}{x_2} \bar{q}_{0i}(x_1) g(x_2) \left\{ \left(-\frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} + \ln \frac{M^2}{4\pi} \frac{(1-z)^2}{z} \right) \left[z^2 + (1-z)^2 \right] + \frac{3}{2} + z - \frac{3}{2} z^2 \right\}$$

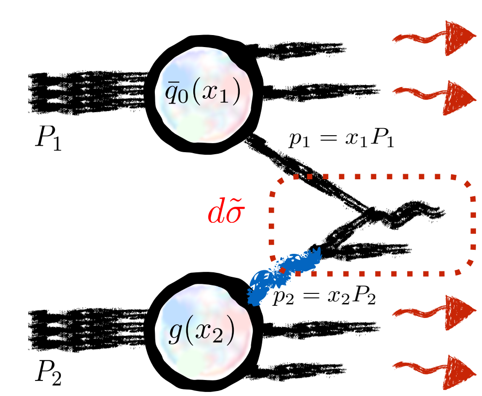
Sum of corrections



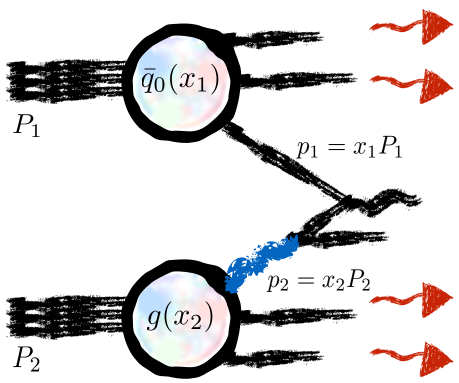
$$d\sigma_1 = -\frac{e^2}{3S}\frac{g^2}{16\pi}\sum_i Q_i^2 \int_{\tau_0}^1 \frac{dx_1}{x_1}\int_{\tau_0/x_1}^1 \frac{dx_2}{x_2}\bar{q}_{0i}(x_1)g(x_2)\left[\left(z^2+(1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}+\ln\frac{Q^2}{4\pi}\frac{1-z}{z}\right\}+6z(1-z)\right]$$

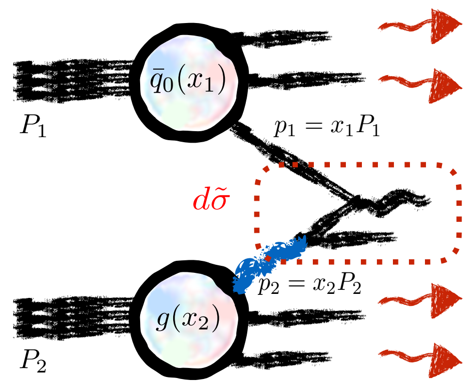
Collinear singularity

$$d\sigma_1 = \frac{e^2 g^2}{48\pi S}\sum_i Q_i^2 \int_{\tau_0}^1 \frac{dx_1}{x_1}\int_{\tau_0/x_1}^1 \frac{dx_2}{x_2}\bar{q}_{0i}(x_1)g(x_2)\left\{\left(-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}+\ln\frac{M^2}{4\pi}\frac{(1-z)^2}{z}\right)\left[z^2+(1-z)^2\right]+\frac{3}{2}+z-\frac{3}{2}z^2\right\}$$



Sum of corrections





We take the sum of this two results

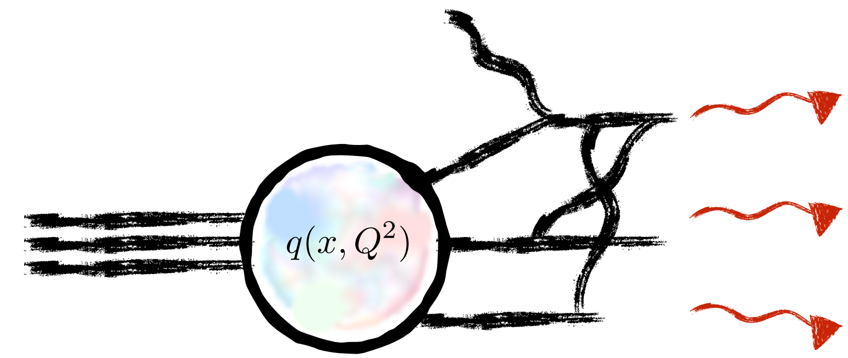
$$d\sigma_1 = \frac{e^2 g^2}{48\pi} \sum_i Q_i^2 \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}_{0i}(x_1) g(x_2) \frac{1}{s} \left\{ \left[z^2 + (1-z)^2 \right] \ln \frac{M^2}{Q^2} (1-z) + \frac{3}{2} - 5z + \frac{9}{2} z^2 \right\}$$

No collinear singularity

$$d\sigma_0 = \frac{\pi e^2}{3S} \sum_i Q_i^2 \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}_{0i}(x_1) q_i(x_2, Q^2) \delta(x_1 x_2 - \tau_o)$$

Renormalized distribution function

Large logarithm



Where does this parameter come from?

We measure this function at a particular scale in DIS

$$d\sigma_0 = \frac{\pi e^2}{3S} \sum_i Q_i^2 \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}_{0i}(x_1) q_i(x_2, Q^2) \delta(x_1 x_2 - \tau_0)$$

$$d\sigma_1 = \frac{e^2 g^2}{48\pi} \sum_i Q_i^2 \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}_{0i}(x_1) g(x_2) \frac{1}{s} \left\{ \left[z^2 + (1-z)^2 \right] \ln \frac{M^2}{Q^2} (1-z) + \frac{3}{2} - 5z + \frac{9}{2} z^2 \right\}$$

This logarithm comes from collinear cross section. It can be large

We get this parameter in the correction as well

We can use distribution measured at any scale but corresponding correction will be different as well

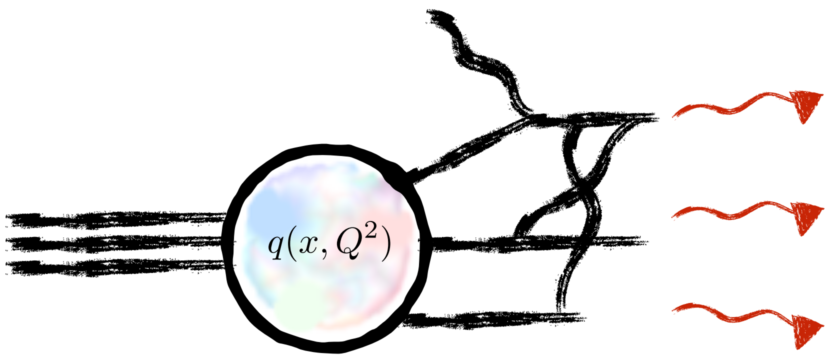
$$\sim g^2 \ln \frac{M^2}{Q^2} \sim 1$$

$$Q^2 \sim 10 GeV^2$$

$$M^2 \sim 4m_\mu^2 \sim 0.1 GeV^2$$

That means we should include all orders of perturbation theory

Large logarithm: what to do?



We need to construct evolution equation

Finite number

$q(x, Q^2)$

Evolve to a different scale

Measure distribution function at large scale

?

$q(x, M^2)$

$\ln M^2/Q^2 \to 1$

Eliminate large logarithms

Evolution equation

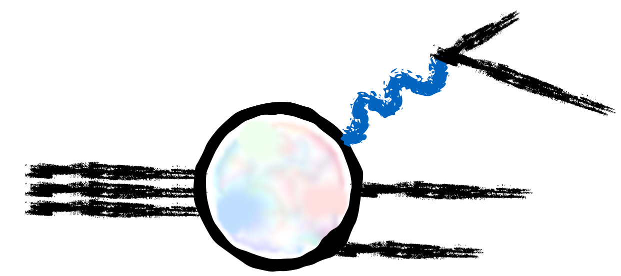
Explicit dependence on the scale parameter

In the first part we calculated

$$q(x,Q^2) = q_0(x) + \frac{g^2}{16\pi^2}\frac{1}{1-\epsilon}\int_x^1 \frac{dy}{y} g(y)\left[\left(z^2+(1-z)^2\right)\left\{-\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} + \ln\frac{Q^2}{4\pi}\frac{1-z}{z}\right\} + 6z(1-z)\right]$$

Apply $\frac{d}{d\ln Q^2}$

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y} g(y)\left[z^2+(1-z)^2\right]$$



Gluon contribution to quark distribution function

Splitting function:

$$P_{qg}(z) = \frac{1}{2}\left[z^2+(1-z)^2\right]$$

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y) P_{qg}\left(\frac{x}{y}\right)$$

Evolution equation

Evolution equation

Evolution equation

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y) P_{qg}\left(\frac{x}{y}\right)$$

We can solve this equation in a form:

Measure from the DIS experiment

Large logarithm

$$q(x,M^2) = q(x,Q^2) + \frac{\alpha_s}{4\pi}\ln\frac{M^2}{Q^2}\int_x^1 \frac{dy}{y} g(y)\left[z^2+(1-z)^2\right]$$

Recall the result for the cross section

$$d\sigma = \frac{\pi e^2}{3S}\sum_i Q_i^2 \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 \bar{q}_{0i}(x_1) q_i(x_2,M^2)\delta(x_1x_2-\tau_0)$$

$$+\frac{e^2g^2}{48\pi}\sum_i Q_i^2 \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 \bar{q}_{0i}(x_1) g(x_2)\frac{1}{s}\left\{\left[z^2+(1-z)^2\right]\ln(1-z)+\frac{3}{2}-5z+\frac{9}{2}z^2\right\}$$

No large logarithm!

Evolution equation

Evolution equation

$$\frac{d}{d\ln Q^2}q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}g(y)P_{qg}\left(\frac{x}{y}\right)$$

Measure from the DIS experiment

Large logarithm

We can solve this equation in a form:

$$q(x,M^2) = q(x,Q^2) + \frac{\alpha_s}{4\pi}\ln\frac{M^2}{Q^2}\int_x^1 \frac{dy}{y}g(y)\left[z^2 + (1-z)^2\right]$$

Recall the result for the cross section

$$d\sigma = \frac{\pi e^2}{3S}\sum_i Q_i^2 \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 \left\{\bar{q}_i(x_1,M^2)q_i(x_2,M^2) + q_i(x_1,M^2)\bar{q}_i(x_2,M^2)\right\}\delta(x_1x_2-\tau_0)$$

$$+\frac{e^2g^2}{48\pi}\sum_i Q_i^2 \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 \left[\bar{q}_{0i}(x_1)g(x_2) + g(x_1)\bar{q}_{0i}(x_2) + q_{0i}(x_1)g(x_2) + g(x_1)q_{0i}(x_2)\right]$$

$$\times\frac{1}{s}\left\{\left[z^2+(1-z)^2\right]\ln(1-z) + \frac{3}{2} - 5z + \frac{9}{2}z^2\right\}$$

No large logarithm!

Evolution equation

The equation has only one large logarithm

Evolution equation

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y) P_{qg}\left(\frac{x}{y}\right)$$

We can solve this equation in a form:

$$q(x,M^2) = q(x,Q^2) + \frac{\alpha_s}{4\pi}\ln\frac{M^2}{Q^2}\int_x^1 \frac{dy}{y} g(y)\left[z^2 + (1-z)^2\right]$$

Let's generalize it!

There are several equations of this kind

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) P_{qg}\left(\frac{x}{y}\right)$$

DGLAP evolution equation(s)

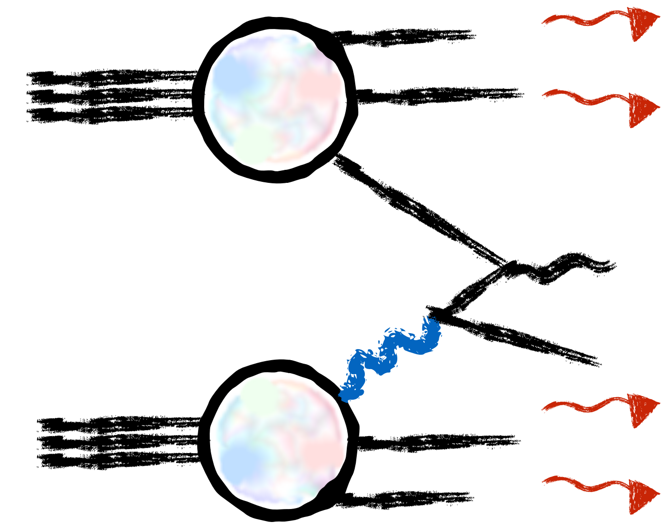
Resummation of $\alpha_s^n \ln^n \frac{M^2}{Q^2}$

V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972)
G. Altarelli, G. Parisi, Nucl. Phys. B126, 298 (1977)
Yu.L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977)

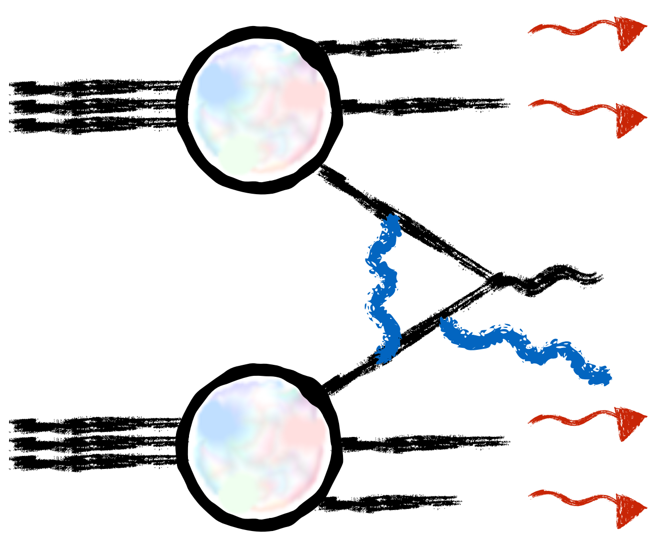
Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

DGLAP equation

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) P_{qg}\left(\frac{x}{y}\right)$$



There is also quark contribution



Outline of the calculation

1) Calculate correction both to quark PDF and parsonic cross section

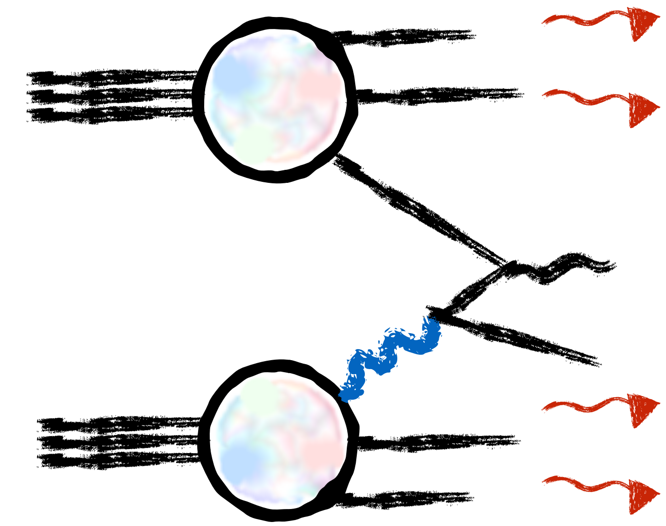
2) Take a sum. Check cancellation of the collinear divergence

3) Obtain correction to the cross section

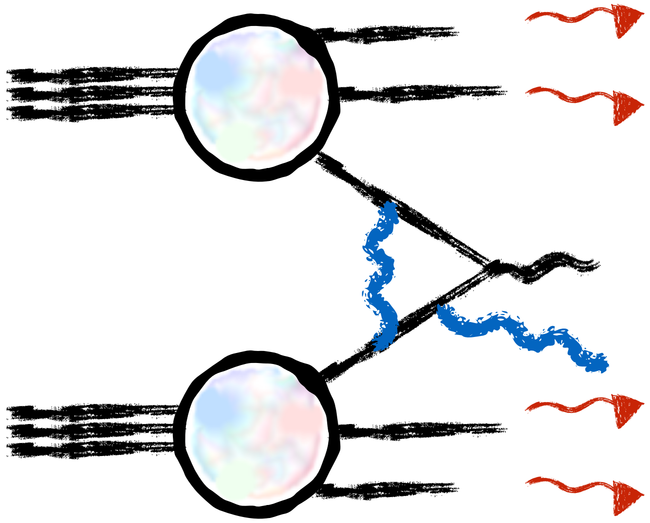
Next slide → 4) Derive corresponding evolution equation

DGLAP equation

$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) P_{qg}\left(\frac{x}{y}\right)$$



There is also quark contribution

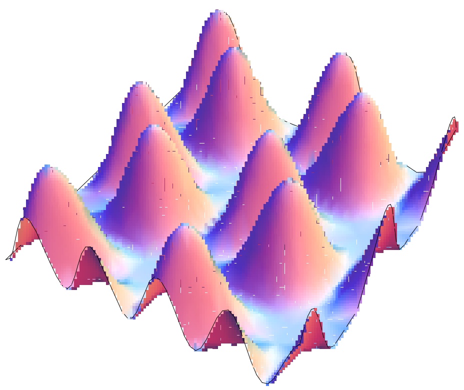


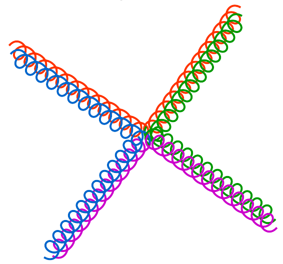
$$\frac{d}{d\ln Q^2} q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2) P_{qg}\left(\frac{x}{y}\right)\right]$$

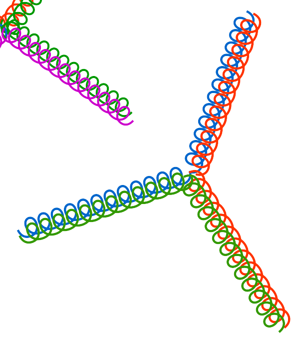
Calculate correction both to quark
PDF and parsonic cross section

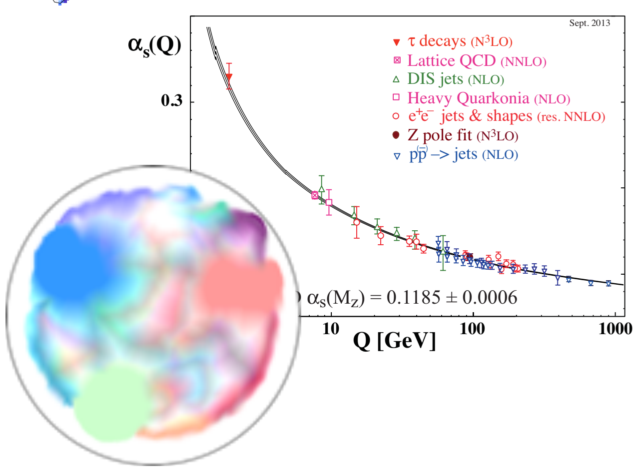
Quark-quark splitting
function

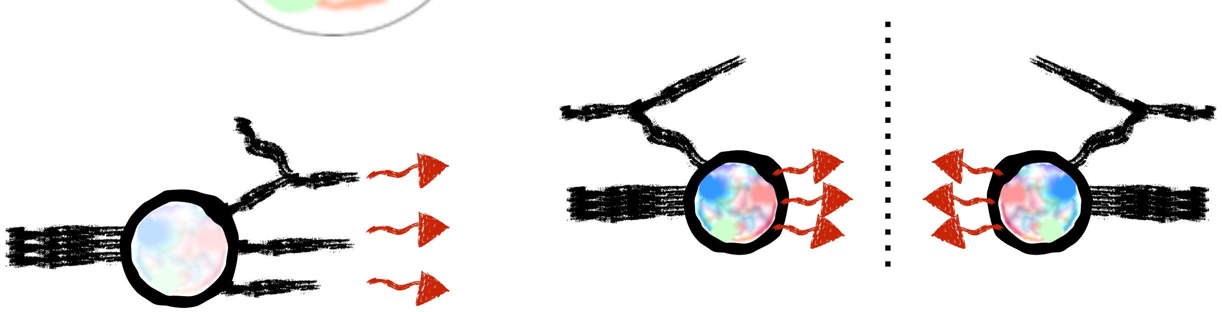
Does it work?

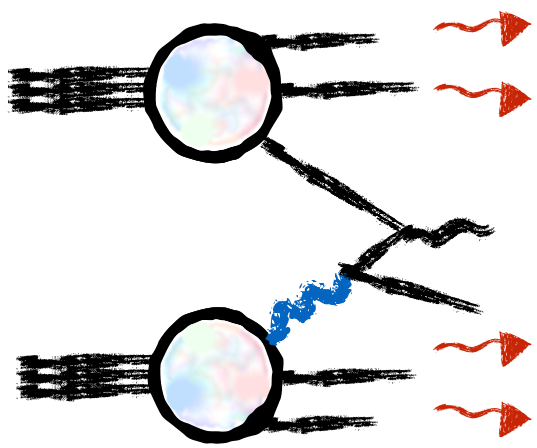


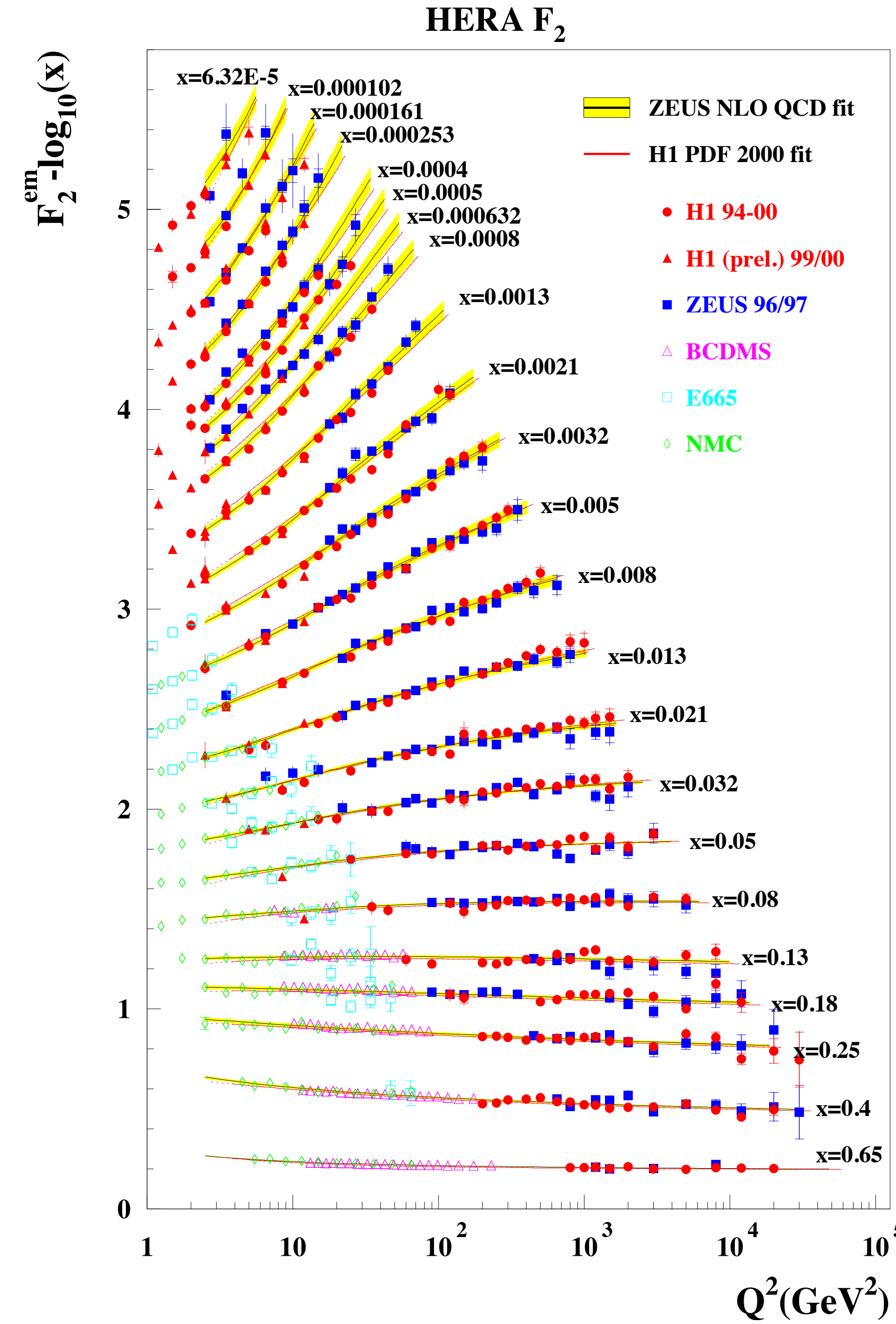












$$\frac{d}{d\ln Q^2}q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right]$$